

# Uncertainty of PIV/PTV based pressure, using velocity uncertainty

Jiacheng Zhang<sup>1</sup>, Sayantan Bhattacharya<sup>1</sup>, and Pavlos P. Vlachos<sup>1,2\*</sup>

<sup>1</sup> School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA

<sup>2</sup> Weldon School of Biomedical Engineering, Purdue University, West Lafayette, IN 47907, USA

\*pvlachos@purdue.edu

## Abstract

Pressure reconstruction from velocity measurements using particle image velocimetry (PIV) and particle tracking velocimetry (PTV) has drawn significant attention as it can provide instantaneous pressure fields without altering the flow. Previous studies have found that the accuracy of the calculated pressure field depends on several factors including the accuracy of the velocity measurement, the spatiotemporal resolutions, the method for calculating pressure-gradient, the algorithm for pressure-gradient integration, the pressure boundary condition, etc. Therefore, it is critical and challenging to quantify the uncertainty of the reconstructed pressure field. The recent development of the uncertainty quantification algorithms for PIV and PTV allows for the local and instantaneous uncertainty estimation of velocity measurement, which can be used to infer the pressure uncertainty. In this study, we introduce a framework that propagates the standard velocity uncertainty defined as the standard deviation of the velocity error distribution through the pressure reconstruction process to obtain the uncertainty of the pressure field. The uncertainty propagations through the calculation of the pressure-gradient and the pressure-gradient integration were modeled as linear transformations, which can reproduce the effects of the spatiotemporal resolutions, the numerical schemes, the integration algorithms, and the pressure boundary condition on the accuracy of the resulting pressure fields. The proposed uncertainty estimation approach also considers the effect of the spatiotemporal and component-wise correlation of the velocity errors in common PIV/PTV measurements on the pressure uncertainty.

The method was firstly validated with synthetic flow fields for the reconstructions by solving the pressure Poisson equation (PPE) and using the least-squares methods (OLS, WLS, GLS) [1]. The synthetic flow fields were generated from a 2D pulsatile channel flow and were contaminated with varying levels of artificial noise that were correlated in space, time, and between components. The root-mean-square (RMS) of the pressure error and uncertainty normalized by the characteristic pressure are compared in figure 1 as functions of the velocity noise level ( $\alpha$ ). For PPE and OLS, the RMS uncertainty matched the RMS error for cases with  $\alpha < 10\%$  but overestimated by about 10% for cases with greater noise. At low noise levels ( $\alpha \leq 5\%$ ), the RMS uncertainty of WLS and GLS reconstructions were underestimated by 40-60%, while the RMS uncertainty was within 10% of the RMS error for the other noise levels. At high noise levels ( $\alpha > 10\%$ ), the RMS uncertainty was overestimated by about 3% for WLS while underestimated by 5% for GLS.

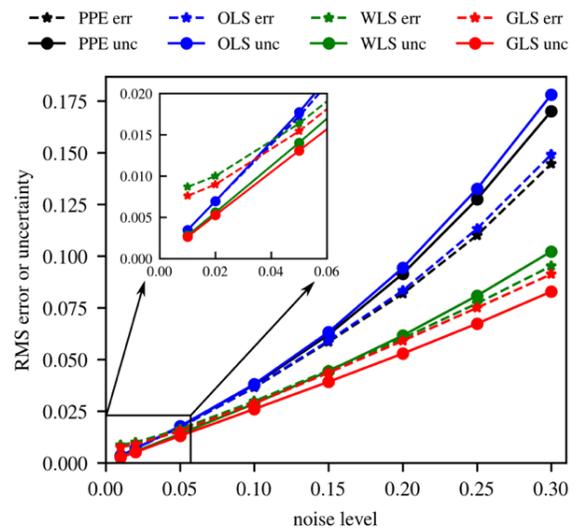


Figure 1 The RMS of pressure error and uncertainty from velocity fields with different noise level

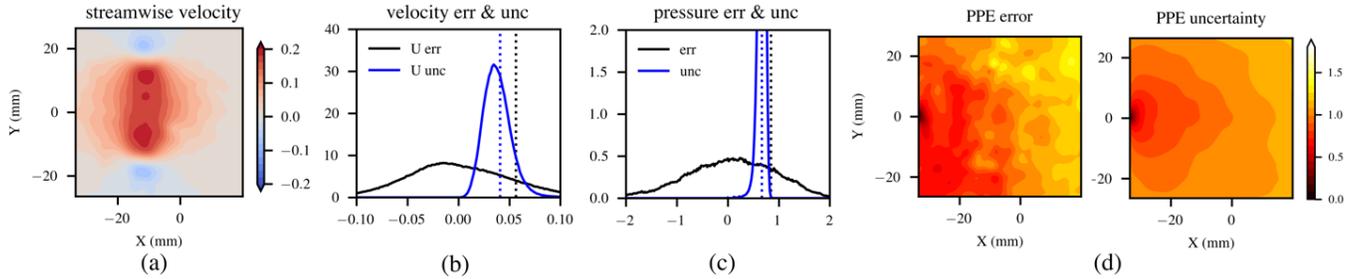


Figure 2 (a) The streamwise velocity field (b) The histograms of velocity error and uncertainty (c) The histograms of the pressure error and uncertainty with the RMS values indicated by the vertical lines. (d) The spatial distributions of the normalized pressure error and uncertainty.

The method was then applied to the experimental velocity measurement of a vortex ring acquired using planar PIV [2]. To account for the effect of the interrogation window overlap on the spatial autocorrelation of velocity errors [3], the autocorrelation coefficient was approximated using a Gaussian function of the spatial separation as:

$$\rho = \exp(\ln(w_{op}) \times r^2), \quad (1)$$

where  $w_{op}$  is the window overlap, and  $r$  represents the spatial separation normalized by the grid resolution. Thus,  $\rho = w_{op}$  between neighboring velocity measurements. The streamwise velocity field is shown in figure 2(a). The histograms of velocity error and uncertainty quantified by the moment of correlation (MC) method [4] are presented in figure 2(b). The histograms of the pressure error and the estimated uncertainty for the reconstruction using PPE are compared in figure 2(c). As indicated by the RMS values, the velocity uncertainty underestimated the velocity error by 27%, while the pressure uncertainty was 21% lower than the pressure error. As suggested in figure 2(d), the pressure uncertainty showed a similar trend across field as the pressure error but was underestimated around the vortex cores, resulting in more uniform spatial distributions.

The method was further applied to the volumetric PTV measurement of a laminar pipe flow [5]. The errors and estimated uncertainties are compared in figure 3(a) for the velocity fields obtained with an Iterative Particle Reconstruction (IPR) based 3D reconstruction [6] and nearest-neighbor tracking, and the pressure fields reconstructed using PPE. As suggested by the RMS values, the velocity uncertainty was 10% higher than the velocity error, while the pressure uncertainty was 7% higher than the pressure error. As shown in figure 3(b), the spatial distribution of the pressure uncertainty was consistent with the pressure error as they were both lowest at the center point of the inflow plane (at  $X=0$  mm and  $R=0$  mm) where the reference pressure was imposed.

Efforts are ongoing to estimate the uncertainty of pressure reconstructed from velocity measurements using volumetric PIV.

## References

- [1] Zhang J, Bhattacharya S and Vlachos P P 2020 Using uncertainty to improve pressure field reconstruction from PIV / PTV flow measurements *Exp. Fluids* **61** 1–20
- [2] Kähler C J, Astarita T, Vlachos P P and Sakakibara J 2016 Main results of the 4th International PIV Challenge *Exp. Fluids* **57** 1–71
- [3] Sciacchitano A and Wieneke B 2016 PIV uncertainty propagation *Meas. Sci. Technol.* **27** 084006
- [4] Bhattacharya S, Charonko J J and Vlachos P P 2018 Particle image velocimetry ( PIV ) uncertainty quantification using moment of correlation ( MC ) plane *Meas. Sci. Technol.* **29** 115301
- [5] Bhattacharya S and Vlachos P P 2020 Volumetric Particle Tracking Velocimetry (PTV) Uncertainty Quantification *Exp. Fluids* **61**
- [6] Wieneke B 2013 Iterative reconstruction of volumetric particle distribution *Meas. Sci. Technol.* **24** 024008

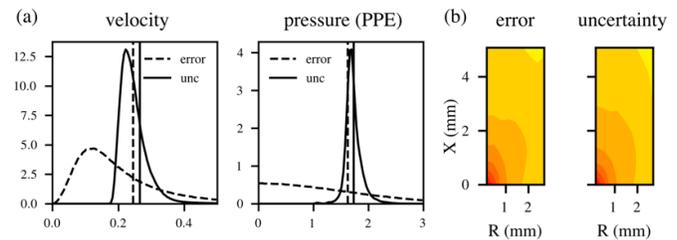


Figure 3 (a) The histograms of the normalized errors and uncertainties in the velocity and reconstructed pressure fields. The RMS values are indicated using the vertical lines. (b) The spatial distributions of the pressure error and uncertainty from the PPE reconstruction.